

# Faddeev Bound State program EFADDY

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Version 1.71: with coupled occupied & transfer states

## Introduction

This program calculates the bound states of two nucleons with a third core. Interactions are by local potentials, with the nucleon wave function orthogonal to set of occupied states in the core. The core may have a set of states of arbitrary energies, spins & parities, and these states may be coupled together by a collective model using a multipole-deformed potential.

## Input Specifications

### HEADER: A80

Card for text describing run.

### FPOT (String in quotes)

Name of file stem: to create FPOT.mout with additional output, and (if NMAX=0) FPOT.spec, FPOT.mel and FPOT.occ with files for STURM.

### AC,ZC,RCORE

**AC** mass of core nucleus

**ZC** charge of core nucleus

**RCORE** r.m.s. matter radius of target nucleus

### NTARG,KTARG,(DEF(I),I=2,5)

**NTARG** abs(NTARG) = number of target states

If NTARG < 0, then the VOLCON logical variable is set TRUE for use by the potential function routines UC(L,R) etc. and read:

**(DEFH(I),I=2,5)** Deformation lengths for 3-body Hamiltonian, as DEF only used for occupied states.

**KTARG**  $K$  projection for coupling states by the rotational model (real)

**DEF(2:5)** deformation lengths  $\delta_2$ ,  $\delta_3$ ,  $\delta_4$ , and  $\delta_5$ . These deformation lengths are found from the fractional deformations  $\beta_K$  by  $\delta_K = \beta_K R$  for nuclear radius  $R$ .

### JTARG(i),PTARG(i),ETARG(i) Card repeated for i=1,NTARG: specification of each target state

**JTARG** spin  $J_c$  (real)

**PTARG** parity  $\pi_c$  (integer +1 or -1)

**ETARG** energy  $E_c$

**Q0CORE** Card read only if maxval(JTARG(:))>0

**Q0CORE** Intrinsic quadrupole moment of core, used for E2 transitions.

**AN,ZN,RN,SN,TT,(ZN2,RSCR)**

**AN** mass of valence particle

**ZN** charge of valence particle

**RN** r.m.s. matter radius of valence particle (0 for nucleons!)

**SN** spin  $s$  of valence particle, 0 or 0.5 (real)

**TT** isospin of valence particle, 0 or 1 (integer)

**ZN2** charge of second valence particle (read in if ZN>0)

**RSCR** screening radius for Coulomb potentials (RSCR=0 is no screening) (read in if ZN>0)

**H2M**

**H2M**  $\hbar^2/2m$  for unit mass  $m$ . Use 20.900795 for AC, AN in amu, and 20.721 for masses in units of neutron mass. With neutron mass units, use 20.748 for the old HH programs and 20.735 for the old CSF programs.

**KMAX,LNCMAX,LNNMAX,SMAX,LLMAX,L2MX,EQN,JNNMX(1:-L2MX)**

**KMAX** Max. hyperharmonic  $K$

**LNCMAX** Max. angular momentum  $L_{cn}$  (between core and neutron) with non-zero potential

**LNNMAX** Max. angular momentum  $L_{nn}$  (between two neutrons) with non-zero potential

**SMAX** Max. spin  $S$  of the two neutrons (integer 0 or 1)

**LLMAX** Max. angular momentum  $L = L_{cn} + L_{nn}$

**L2MX** Absolute value is max. angular momentum  $\lambda$  between the interacting pair and the spectator.

**EQN** (character in quotes) Type of equation to solve: 'F'=Faddeev, 'T'=T-Schrodinger:HH, 'Y'=Y-Schrodinger.

If 'a' lle EQN lle 'z', then set FESH=true for Feshbach reduction.

**JNNMX(1:-L2MX)** Max. angular momentum  $J_{nn}$  of valence pair, for each  $L$  (i.e. read only if  $L2MX < -1$ ).

If  $KMAX < -1$ : **K3MAX(L,IC),L=0,MAXLR+1**

Max. hyperharmonic  $K$  for each particular  $L=L_{nn}$  and IC, followed by max.  $K$  for both  $L_{nn} \geq 2$  and  $L_{(nn)c} \geq 2$ , where  $MAXLR = \min(MAXL, 3)$  with  $MAXL = \max(LNNMAX, LNCMAX, L2MX)$ . This card is repeated for each IC=1 to NTARG.

If  $K3MAX(0,1) < -1$ : **K2MAX(L,IC),L=0,MAXLR+1**

Max. hyperharmonic  $K$  for each particular  $L=L_{cn}$  and IC, followed by max.  $K$  for both  $L_{cn} \geq 2$  and  $L_{(cn)c} \geq 2$ .

This card is repeated for each IC=1 to NTARG.

If  $KMAX = -1$ : **PWFILE, THRESH**

**PWFILE** (string in quotes) name of file of partial wave weights from previous calculation.

**THRESH** real positive threshold for including partial waves in this calculation.

*At present, the KMAX=-1 option only works for EQN='F'*

If FESH: **Kmaxf(1:NTARG),EFesh,RLOC,NDROP**

**Kmaxf(1:NTARG)** Perform Feshbach reduction to  $K_{\max}$  for each target state.

**EFesh** Feshbach target eigenenergy

**RLOC** Radius for local Pauli blocking

**NDROP** Number of adiabatic energy surfaces to drop for Pauli Blocking.

**RR,NLAG,NJAC,RINNER**

**RR** Scaling factor for hyperradius grid

**NLAG** Number of hyperradial grid points.

If  $NMAX \neq 0$ , use NLAG Gauss-Laguerre quadrature points, and perform diagonalisation for eigen-energies

If  $NMAX = 0$ , use NLAG *regularly spaced* hyperradial points from RR to  $RR*NLAG$ , and write the output files FPOT.spec, FPOT.mel and FPOT.occ with files for STURM

**NJAC** Number of Gauss-Jacobi hyperangular quadrature points.

**RINNER** Radius for orthonormalisation of Pauli Projection operators

If  $NDROP > 0$ : **NJT, JTOTS(i), PARITIES(i), i=1, NJT**

**NJT** Number of angular momentum/parity sets (integer)

**JTOTS(i)** Total angular momentum (real)

**PARITIES(i)** (character in quotes) Parity ('+' or '-')

If  $NDROP < 0$ : **NJT, (JTOTS(i), PARITIES(i), DROPS(i), i=1, NJT**

**NJT, JTOTS, PARITIES** As above

**DROPS** Number of adiabatic energy surfaces to drop for Pauli Blocking for each spin/parity combination given.

**NMAX, EMIN, DE, EMAX, MEIGS, MOMDIS**

**NMAX** Number of Legendre-polynomial basis functions for diagonalisation.

If  $NMAX \neq 0$ , use NLAG Gauss-Laguerre quadrature points, and perform diagonalisation for eigen-energies

If  $NMAX = 0$ , use NLAG *regularly spaced* hyperradial points from RR to  $RR*NLAG$ , and write the output files FPOT.spec, FPOT.mel and FPOT.occ with files for STURMXX

**EMIN** If  $NMAX > 0$ , energy for inverse iteration: find eigenstates nearest to EMIN. Find MEIGS states.

If  $NMAX < 0$ , find all eigensolutions. MEIGS must be large enough.

**DE** (Read if  $NMAX < 0$ ) Energy step for discrete response functions.

**EMAX** (Read if  $NMAX < 0$ ) Maximum energy for discrete response functions.

**MEIGS** Number of eigensolutions to find.

**MOMDIS** For each solution calculate the spherical & projected momentum momentum distributions, using momentum up to  $k_{max} = 2 \text{ fm}^{-1}$ .

**DX, XMAX, DY, YMAX, RNODE, ITRBS, LBSMAX, KAPP, {EPS2, N3BLOCK}**

**DX** step size for bound states

**XMAX** max. radius for bound state wave functions.

**DY** spacing of splines for  $y$  dimension of each blocked state

**YMAX** maximum  $y$  of each spline set.

**RNODE** max. radius for counting nodes in bound state wave functions.

**ITRBS** Non-zero to print bound state details (e.g. odd for wave functions).

**LBSMAX** Maximum  $l$  for bound-state wave functions.

**KAPP** Non-linearity parameter for spline functions in Pauli blocking (if zero, use 1d-3)

**EPS2** {read if KAPP<0}: threshold norm of Pauli projection operators, (default value is 0.1)

**N3BLOCK** {read if KAPP<0}: number of 3-body bound states to read in 51, to give blocking projection operators (default value is zero).

**BKIND,NV,LV,JV,BE,START,NOM**

Occupied Y states  $\sum_{l,j,c} |(sl)j, I; J\rangle$  for  $s = \text{SN}$  and core state  $I = \text{JTARG}(c)$ :  
or occupied T states  $\sum_{l,S} |(ss)S, l; J\rangle$  for  $s = \text{SN}$ :  
card repeated until the following line of zeros is read.

**BKIND** Component kind: 1 for  $\beta$  (nucleon-core=Y), 2 for  $\alpha$  (NN=T) occupied states.

BKIND=3 or 4 (resp.) for transfer states.

BKIND=5 or 6 (resp.) for continuum bin states (unoccupied!).

If BKIND < 0, use |BKIND| as a test wavefunction only (not occupied).

If BKIND > 10, then use BKIND-10 (and BKIND+10 if BKIND < -10) and also, if searching for potential scaling factor, save this factor in array VSCALE(mod( $l,2$ )) for use by potential function UC(L,R) for e.g. rescaling the central potential to depend on the parity of the ground state. Also read in DE,NK,ISC

**NV** Number  $n$  of nodes (including origin) in first component with  $l = \text{LV}$  (except if NOM < 0).

If NV < 0, force unbound state search.

For continuum bins, the incoming channel is number NV.

**LV** An  $l$  value in the bound state:  $(-1)^l \times \pi_1$  gives overall parity

**JV** BKIND=1,3: Total angular momentum  $J$  in state  $\sum_{l,j,c} |(sl)j, I; J\rangle$  (real)

BKIND=2,4: Total angular momentum  $J$  in state  $\sum_{l,S} |(ss)S, l; J\rangle$  (real)

**BE** If positive, use this binding energy, and rescale the potential;

if zero, find binding energy for fixed potential;

if negative, search for unbound state giving phase shift  $\delta = 90^\circ$ .

**START** Initial value for binding energy or potential multiplier

**NOM** If NOM < 0, make NV the number of nodes in channel number |NOM|, and ensure that this channel is non-zero.

If NOM > 0, read in list of omitted partial waves:

one per new card in format (integer, real, integer):  $l, j, c$  ( $c$  = core state index).

**If BKIND=5 or 6** Also read a card with:

**DE** Width of bin in MeV: from |BE|-DE/2 to |BE|+DE/2

**NK** Number of  $k$  values in quadrature for bin.

**ISC** Scaling weight factor before quadrature:

2:  $\exp(-i\delta(k))$  4:  $\sin(\delta(k)) \exp(-i\delta(k))$  12:  $k \exp(-i\delta(k))$  14:  $k \sin(\delta(k)) \exp(-i\delta(k))$

(repeated until BKIND=0)

0 0 0 0 0 0

## NN and NC potentials

These are specified by including FORTRAN functions for:

**NN interaction:** VC(L,R), VLS(L,R), VT(L,R), VLL(L,R) where VLS is the  $(\mathbf{s}_{n1} + \mathbf{s}_{n2}) \cdot \mathbf{L}$  interaction;

**NC interaction:** UC(L,R), ULS(L,R), ULSC(L,R), ULL(L,R), UT(L,R) where ULS is the neutron spin-orbit  $\mathbf{s}_n \cdot \mathbf{L}$  and ULSC the core spin-orbit  $\mathbf{J}_c \cdot \mathbf{L}$  potential.

VT and UT are tensor potentials using spins  $(\mathbf{s}_{n1} + \mathbf{s}_{n2})$  and  $(\mathbf{s}_n + \mathbf{J}_c)$  respectively, and do not change the core state  $\mathbf{J}_c$ .

When any  $\delta_K \neq 0$ , the central interaction UC(L,R) is deformed by all the non-zero  $\delta_K$  according to

$$V_L(r, \theta) = \text{uc}(\mathbf{L}, r - \sum_{K=2}^5 \delta_K P_K(\cos \theta) \sqrt{\frac{2K+1}{4\pi}})$$

and projected by 9-point Gaussian quadrature onto  $K = 0$  and those multipoles  $K \geq 2$  for which  $\delta_K \neq 0$ . The resulting  $V_L^K(r)$  are used to couple core states  $J_c, J'_c$  for which  $\Delta(J_c, J'_c, K)$  holds, according to a collective model. The value of  $\mathbf{L}$  is given by the angular momentum between the particle and the core in the partial wave being fed by the interaction.

If all the **JTARG** values are zero, then the quadrupole deformation length is used to construct a form factor for monopole excitation couplings.

**NC interaction outside RINNER:** UUC(L,R), UULS(L,R) replace UC and ULS.

**Three-body interaction:** v3B(RHO,...), u3B(RHO,...) for NN and NC partitions respectively.