Book Proposal

Version 3i, 20th July.

Title: "Nuclear Reactions for Astrophysics"
Subtitle: "Principles, calculation and applications of low-energy reactions"

Submit to Publisher: Cambridge University Press

Reasons for writing:

1. Approached by CUP to consider a book on exotic and radioactive nuclei and astrophysical applications.
2. We realise that there is a need for an introductory graduate text on the theory of low-energy nuclear reactions as they occur in stars, and as the same nuclei are probed in modern experiments with radioactive nuclear beams
3. Need to summarise and explain practical calculation methods.

Length: ~ 300-400 pages (as prepared in Latex)

Number of illustrations: ~ 100 - 150 original line drawings/graphs (in postscript)

Date of Completion: First draft: end of 2006, final version: summer 2007

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Level and Readership:

1. Primarily aimed at beginning graduate students in nuclear physics and in astrophysics, who have already some knowledge of angular momentum couplings.
2. Second, physicists, especially academic staff and student nuclear experimentalists, who find they need direct-reaction and R-matrix theory to analyse their experiments.
3. The first two chapters will be useful for final-year undergraduates in nuclear astrophysics (e.g. at Surrey), and as a supplementary text for astrophysics students.
4. International physicists can use this as a reference work for coupled channels theory when using the FRESCO code.

Some of the material will be presented and tested on graduate students and nuclear experimentalists before the final version is settled.

Letters of Support
Since raising the possibility of this book with my international colleagues, we have had many supportive and enthusiastic emails of support for this proposal. Some want the book now.

Comparisons with Competing Books:

1. *Claus E. Rolfs and William S. Rodney*, "Cauldrons in the Cosmos", University of Chicago Press, 1988.
   This has been a standard textbook for many years, but is no longer in print. Discusses in detail astrophysics and experiments, but no nuclear reaction theory beyond simple partial widths and penetrabilities. It assumes cross sections or partial widths are known (by measurement or calculation), so the present proposal is a necessary supplement.

2. *Ettore Gadioli, P.E. Hodgson*, "Pre-equilibrium Nuclear Reactions", Oxford Studies in Nuclear Physics: Clarendon, 1992.
   This book gives a comprehensive account of the experimental and theoretical research that has been devoted to the study of pre-equilibrium reactions. Only the historical introduction describes reaction theory in detail, before mainly dealing with multistep compound and multistep direct theories. The entire focus is on medium and high energy reactions, with nothing about reactions at astrophysical energies.
3. *Y. Suzuki, R.G. Lovas, K. Yabana and K. Varga* "Structure and Reactions of Light Exotic Nuclei", Taylor & Francis, 2003.
   The reaction part of this book discusses only high-energy Glauber and semiclassical methods, and the structure part gives details of cluster model approaches to p-shell nuclei. There is no mention of astrophysics, low-energy reactions, R-matrix theory, data fitting, or transfer spectroscopy. The only overlaps are with parts of our proposed chs. 13 and 14.
4. *C.A. Bertulani*, "Introduction to Nuclear Reactions", Institute of Physics, 2004.
   This covers all kinds of reactions: includes heavy-ion, fission and relativistic theory, but mentions eg R-matrix methods only on 3 pages. No numerical methods or data fitting.
5. *Pierre Descouvemont*, "Theoretical Models for Nuclear Astrophysics", Nova Science Publishers, 2004.
   This short book started as a review article that became too long. Contains much useful material, but it is less a textbook than a set of research notes with some introductory material added.

How this book differs from others available:
The proposed book has a unique exposition of calculation methods and practice, as well as of general theory. This should 'complete the circle', and encourage e.g. many students and experimentalists to use and rely on this kind of approach. An underlying 'theme' will be of a coupled-channels formalism, which allows strong and electromagnetic interactions to be brought under one roof. The unique chapters 7, 8, 13 and 15 have never before appeared in textbook form.

Discussion about the title:
It should be emphasised that
(a) this book does not contain advanced nuclear astrophysics itself, but the nuclear ingredients *for* astrophysics, and
(b) the applications of this theory are not *only* in astrophysics, but also in nuclear reactors, in materials diagnostic accelerators, and in accelerator-driven transmutation systems now being proposed.

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Part I: Introduction to nuclear astrophysics

1. Survey of reactions of nuclei
    a. Strong, electromagnetic and weak processes
    b. Types of reaction outcomes and Q values.
    c. Cross sections: definition & units.
    d. Laboratory and centre-of-mass: energies, angles and cross sections
    e. Energy and time scales for direct and compound reactions
    f. Low-energy behaviour and astrophysical S-factors. Neutrons.
    g. Maxwell-Boltzmann distributions for absolute and relative velocities.
    h. Thermal reaction rates $\langle \sigma v \rangle$ and the Gamow peak.
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    Identifying particular reactions in
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    b. Helium burning
    c. Heavy elements up to iron
    d. Nucleosynthesis beyond iron
    e. Primordial nucleosynthesis
    f. Abundances and anomalies
    g. Need for experiments (direct & indirect) and theoretical models

Part II: Low-energy Reactions

Prologue: Method of theoretical and calculational exposition using FRESCO .
Sections with * will tell how to use FRESCO for practical calculation of typical astrophysical reactions (see 'computer software' above, for more details).

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Appendix A. Getting started with FRESCO
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References
Notation
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What is not included (only mentioned, with references):

1. Microscopic models: RGM, shell model, mean-field models, RPA
2. Heavy-ion fusion
3. Nuclear matter, equation of state, neutron stars
4. Neutrino-induced reactions
5. Statistical compound-nucleus reactions
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    b. doorway states
    c. reactions on heavy nuclei with high level density
    d. on-shell approximations to multistep direct processes

Computer software
I want to include a computer CD with the book, at least with the first edition, with a standard copy of my coupled-channels program FRESCO, so that calculational examples (inputs and outputs) on the CD can be referenced from the book. This will allow the material presented to have immediate practical applications, and extends the range of exercises that may be usefully set.

A website www.fresco.org.uk (hosted at Surrey) will also be available for more up to date information, for manuals, and help with running the program. The CD is necessary as well, in order to have a fixed program as a reference point for publications using FRESCO.

* The sections marked with an asterisk will contain a distinct subsection (eg in a box, or smaller font), which will have (typically):

1. Description of model space, Hamiltonian and coupling scheme
2. File names for FRESCO input and output
3. Criteria for accurate calculation
4. Plot of important results
5. Description of connection between the model details and output

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Numerical Methods
There will be little discussion of numerical methods as such, but just enough general guidelines that accurate results can be obtained, and more emphasis on the transparent connection between model assumptions and the results obtained.

Summaries/abstracts for each chapter

Part I: Introduction to Nuclear Astrophysics

1. **Survey of reactions of nuclei**: We begin this book by a very general introduction of reactions that involve nuclei, mentioning the different types of reactions that happen in nature and including generic examples.

This chapter starts by (a) the identification of the reactions that proceed by different interactions, namely strong, electromagnetic and weak processes, emphasizing in a qualitative way the differences between them, and follows (b) with the presentation of the different types of reaction outcomes with a pictorial representation: elastic and inelastic scattering, transfer (stripping and pick up), radiative capture, beta decay, etc. The Q-values are defined in terms of mass excess.

The concept of cross section (c) is then introduced, and its definition and units are given.

Laboratory and centre of mass frames are discussed in (d) and the relations between energies, angles and cross sections in those frames are deduced.

Next (e), two different types of mechanisms are mentioned and shown with diagrams, namely the direct and the compound nucleus mechanisms, and comparisons in energy and time scales shown with examples.

The low energy regime is then the focus of section (f) and the behaviour of cross section at very low energy is identified with some examples, showing some plots of cross sections and the astrophysical S- factor defined here. Neutrons, as particles with no charge, are treated separately.

In (g) the general characteristics of thermonuclear reactions as a source of nuclear energy are explained. The concept of stellar reaction rate at a certain temperature is introduced. The Maxwell-Boltzmann velocity distribution is presented and the reaction rates for a generic pair of particles are calculated.

The behaviour of the stellar rates for neutron and charged particles induced reactions are presented (h) as a function of energy, and the Gamow peak identified.

2. **Nuclear reactions in stars**: The aim of this chapter is to identify specific reactions that are present in astrophysical environments, pointing out particular features and showing, when appropriate, plots of experimental data. References are given to later sections, where methods and examples to calculate these reactions will be presented.

We start (a) by addressing hydrogen burning in the three proton-proton chains, continuing with the CNO cycle including the by-cycle and additional cycles, the hot CNO cycle leading to the rp process. The NeNa and MgAl cycles are also mentioned.

The helium burning is presented in (b), starting with the triple alpha process, followed by heavier element α-burning via (α,γ) reactions, and neutron production in reactions induced by alpha particles. Red giants will be mentioned in this context.

The presentation continues (c) by the burning of heavier elements in the nucleosynthesis of massive stars, from carbon burning to silicon burning, until iron. White dwarfs and neutron stars will be discused as possible evolutions of a star, as well as the explosive burning leading to supernovae.

The nucleosynthesis beyond the iron is mentioned next (d). Neutron capture reactions are described as the basis for the mechanism of the s and the r- processes, and paths in a chart of nuclei are shown.

Primordial nucleosynthesis is considered (e) by the identification of the Big-Bang reactions. Transfer, radiative capture induced by light ions (n, p, d and α) as well as charge exchange (n,p) reactions are mentioned. A diagram of the path of primordial reactions from neutrons to ^{7}Be is shown, as well as plots of S-factors for a few examples.

The question of abundances is addressed (f) by comparing observed values with those calculated from primordial nucleosynthesis. Isotopic anomalies are identified, and we outline in particular the origin of light elements Li, Be and B by cosmic rays and spallation.

The final point (g) is the need to understand the details of the reactions measured, as will be evident as more accurate data are obtained, and the need to have ways of theoretically modelling the reactions in order to produce reliable extrapolations to the energies of astrophysical interest.

Part II: Low-energy Reactions

3. **Scattering Theory**: This chapter develops basic scattering theory, building on previous courses on angular momentum, and develops the framework to describe scattering in astrophysics and other applications.

It begins with (a) single-channel scattering from a spherical potential with a partial-wave expansion, presenting phase shifts and Coulomb+Nuclear scattering. Graphs of typical behaviour, with more details given in 6a.

Then (b) the foundations for multi-channel scattering are formed for two-body partitions, using the spin and excitation energies of projectile and target, coupled in either *LS* or *jj* coupling orders, leading to a scattering *S*-matrix.

These basis states then enable (c) a coupled channels set of equations to be formed for the radial wave functions, and boundary conditions defined. The meanings of Hermiticity and unitarity are discussed, with examples for real and optical potentials. Diagonal and off-diagonal couplings are distinguished, and also local and non-local interactions.

In section (d) the solutions of these coupled equations are described equivalently by standard Green function techniques, leading, via the two-potential formula, to plane-wave and distorted-wave series solutions.

Section (e) shows how to find the scattering of identical particles in entrance and exit channels.

Resonant solutions are discussed in (f), both as Breit-Wigner expressions for isolated resonances, and as poles of the *S*-matrix. Poles on the negative imaginary *k*-axis are classed as virtual states, and the wave functions for both poles and virtual states are calculated and plotted.

The theory of one-photon channels is then developed (g), starting from Maxwell's equations for **E** and **B**, via the vector potential **A** in the Coulomb gauge, to the combination of photon channels within the coupled-channels set using current operators.

Finally, polarisation theory is presented (h), showing how polarised and aligned beams are defined, and how reactions with non-zero analysing powers then lead to discriminating cross sections for detectors at different scattering angles.

Notes.

1. Partial wave expansion of Coulomb-distorted plane wave is stated, not derived.

2. Functions *F* and *G* are used even for neutral scattering, to avoid introducing separate spherical Bessel functions.

3. Formal scattering theory (eg. for on- and off-shell $T(\mathbf{k},\mathbf{k}')$) is *not* included: as only partial-wave Green functions are used.

4. **Reaction Mechanisms**: This chapter finds the couplings between channels in terms of the collective or single-particle properties of the interacting nuclei.

Inelastic couplings (a) from deformed or vibrating nuclear shapes, where the potential depends on the distance to the surface. Both nuclear and Coulomb couplings are derived. Then single-particle couplings between states of a nucleon (or cluster of nucleons), derived from a multipole expansion of summed nucleon and core potentials with the interacting partner nucleus. Numerical and graphical examples.

Transfer couplings (b) occur when the nucleon or cluster moves over to the partner nucleus. Post and prior forms of the channel Hamiltonian are used to derive the non-local transfer couplings, including remnant terms. Conditions are given for the zero-range approximation, its correction by the local energy approximation, and what these give for sub-Coulomb transfer reactions typical in astrophysics. A numerical example will compare the FR, ZR and LEA methods in low-energy transfer reactions. Finally, we discuss the relative roles of spectroscopic factors and asymptotic normalisation coefficients in determining cross sections.

Knockout reactions (c) such as (p,α) may occur by transfer of a heavy core or transfer of a light cluster, and these add coherently via identical-after-exchange exit channels. The accuracy of including e.g. just the V(pα) matrix element will be determined.

Charge exchange reactions will be modelled in (d) by defining Fermi and/or Gamow-Teller matrix elements for local couplings.

Finally, section (e) will show how single-photon electric and magnetic couplings may be derived for the coupled-channels framework, starting from current operators and deriving charge operators via the Siegert theorem.

Note:

1. Reduced matrix elements will be defined explicitly and evaluated in detail. The resulting couplings using these in *jj* coupling order will be given, but not derived.

5. **Connecting Structure with Reactions**: This chapter shows how the collective and/or single-particle properties of chapter 4 may be derived from microscopic nuclear models.

Section (a) briefly summarises the range of microscopic structure models that may be used: the shell model, the RGM, cluster models, and mean-field approaches.

The collective properties discussed in section (b) are the overall size and shape. Their relations to rms radius, fractional deformation, deformation length, spectroscopic and intrinsic quadrupole moments are given. Folded potentials are constructed just using these collective properties in conjunction with some effective NN interaction.

Section (c) shows show to derive single-particle properties from antisymmetric microscopic models by means of fractional parentage overlaps of nuclear wave functions. The importance of defining and transforming phase conventions and normalisations is emphasized. The absolute values of fractional parentage coefficients then allow spectroscopic factors to be microscopically derived, and sum rules for these coefficients are given. Finally, the dynamical effects of short-range correlations on spectroscopic factors are briefly discussed.

The next section (d) repeats this overlap procedure for cluster overlaps, and then presents some simple models for using Pauli blocking models to estimate cluster overlaps.

Charge-exchange overlaps are defined (e) for nuclei of the same atomic mass number, for both Fermi and Gamow-Teller operators. Their contributions to weak-interaction β-decay lifetimes are given.

Finally (f), generalised nuclear and Coulomb matrix elements are defined between any pair of states of the same nucleus. The Coulomb elements are related to $B(Ek)$ values and to gamma-decay lifetimes.

Notes:

1. Different effective NN interactions (M3Y, JLM, etc) will be mentioned, referenced, and used in numerical examples, but the details of the fitting in their derivations will not be discussed.

2. There will be no numerical examples using microscopic models, only references, for reasons of presentational simplicity. Rather, this chapter establishes a 'common language' between structure and direct-reaction theories.

6. **Solving the equations**: Having defined the coupled equations in ch. 3, and their couplings in chs. 3 & 4, this chapter shows how to solve them exactly and under various approximations, and the physical significances of these approximations.

Section (a) demonstrates the simplest one-channel solutions for scattering in the presence of nuclear and Coulomb potentials by means of radial integration and matching to Coulomb functions asymptotically. Integrated cross sections are shown and interpreted for neutral (neutron) scattering, and also scattering from complex optical potentials. The typical behaviours of the elastic S-matrix element and phase shifts will be shown.

Section (b) shows how to solve N coupled channels equations for local couplings, by means of N linearly independent solutions, and then goes on to elucidate the meanings of weak, strong and multi-step couplings by connection with the series solutions of 3(d). There is a need for iterative methods when non-local (e.g. transfer) couplings are present.

Simplifications are discussed in (c), starting with the feasibility of one and multi-step DWBA solutions, if effective (optical) potentials are known for some initial set of channels. Transfer non-orthogonalities also simplify for the first and last steps, and hence very usefully for one-step reactions. Finally, transformations of the coupled channels set are shown to handle long-range couplings, by matching at a smaller radius to coupled asymptotic wave functions, and to transform non-local electromagnetic couplings to local form, by using the long-wavelength and weak-coupling approximations for the photon channel.

The coupled equations may also be solved (d) with expansion over a square-integrable basis, the most useful of which is the R-matrix basis defined by a logarithmic-derivative boundary condition in each channel. Derivations are given for the R-matrix, both as a sum over R-matrix poles and as a solution of linear equations, and hence for the scattering S-matrix. The convergence of the one channel wave function is shown in the R-matrix basis, and its rate of convergence demonstrated. The use of real or complex Buttle corrections to improve accuracy is demonstrated.

Note:

1. There will be little discussion of numerical differential-equation methods as such, but just enough general guidelines that accurate results can be obtained under whatever approach may be chosen.

7. **R-matrix phenomenology**: Since scattering results depend only on the asymptotic S-matrix, and this depends on the R-matrix which can be written as a sum of R-matrix poles, a popular phenomenological approach is to fit scattering data by suitably chosen R-matrix poles. This chapter demonstrates this process, and also its combination with potential scattering in a hybrid model.

Section (a) presents the R-matrix formalism, first for one-channel scattering, using Lane and Thomas [3] as a reference for theory not already described, showing the connection between the Green function and the R-matrix. The weak-coupling limit is defined for the two-channel case, and then the connection to the common simplified parameterisation of R-matrix poles. Interferences between poles are illustrated.

The interpretation of R-matrix poles is discussed in section (b), starting with an isolated pole and its Breit-Wigner formulation, so the connections between formal and observed positions and widths may be derived. The role of 'background poles' to simulate non-resonant effects is elucidated. The formal-observed connections are generalised for multiple poles (multiple resonant and/or background poles). The dynamic effects of sub-threshold (bound) states is shown, and the connection between R-matrix and ANC parameters.

Finally, a hydrid approach is demonstrated in (c), where the R-matrix from a (coupled) potential model has added to it phenomenological terms to describe specific resonances as required.

Note:

1. The Lane & Thomas theory is generalised systematically to transfer and photon channels, so the R-matrix will not generally be symmetric.

2. The notations of Lane & Thomas will be used where feasible, and where not, explicit connections will be given to their expressions. For example, we give the connections between our S and their U matrices, and to their W matrix.

8. **Fitting Data**: This chapter describes how potential, coupling, spectroscopic and R-matrix parameters may be determined by fitting known data, and then used to extrapolate to regions of astrophysical interest, in particular to low-energy astrophysical S-factors.

This fitting begins (a) with optical potentials for elastic scattering, by determining parameters of Gaussian or Woods-Saxon parametrisations. Common ambiguities are discussed. Fitting within coupled channels (b) gives separate bare and optical potentials, when couplings are added, or not added, respectively before comparison with data.

Fitting inelastic cross sections to discrete collective states (c) often allows extraction of separate Coulomb $B(Ek)$ strengths and nuclear deformations. Fitting transfer cross sections is introduced in (d), and discussed further in a later chapter. When polarisation observables are available (e), fitting should also include spin-orbit and possible tensor potentials.

The most systematic approach to fitting data is (f) to minimise the total χ^2 from summed squared differences between theory and experiment, divided by the experimental variances. Statistical and systematic experimental errors have first to be treated differently. χ^2 fitting is demonstrated numerically for the various fits discussed earlier in this chapter, and then fitting of R-matrix parameters is discussed. Some general hints are presented for the initial and final stages of χ^2 fitting, and the possibility of systematic improvement of parameters is emphasized. Methods for the estimation of errors in fitted parameters are given, along with guides for the errors in extrapolated predictions. Some special features of MINUIT fitting are discussed, as this is used in the numerical examples, especially the treatment of bounded parameter variations. Finally, section (g) discusses the peculiarities of fitting single or multiple resonances, both with potentials and within R-matrix phenomenology.

Note:

1. Specific minimization strategies (steepest descent, simplex, simulated annealing, etc) are not discussed, only the general features of using MINUIT (which includes these as options) within FRESCO.

Part III: Nuclear Input to Stellar Models

9. **Network calculations**:

We consider (a) hydrostatic equilibrium processes, starting with the definition of stellar models. Knowing the overall density, temperature and isotopic ratios of each species, and having information about the reaction rates of the different nuclear reactions that are part of the scenario, we derive the set of coupled equations to determine the time variation of the abundances of each isotope, as well as the energy production. This section ignores inhomogenieties and any fluctuations of the medium, so can be considered as a starting point for any stellar calculation.

In (b) non-equilibrium processes, that include the Big Bang and the explosive stellar environments like supernovae, are outlined. The features of primordial nucleosynthesis are discussed in detail in (c), including photon and neutrino evolution. Reference is given to the standard code NUC123 of Kawano [4].

10. **Improving the accuracy**: Nuclear physics inputs are essential features that have to be used in network calculations. We want to show how to relate the uncertainties connected with this input and those of the final results. We outline in (a) the sensitivity of abundance determinations and energy production estimates to the uncertainties in the nuclear cross sections. We also discuss (b) the uncertainties that arise in the extrapolation from experimental to astrophysical energies, comparing polynomial fits to theoretical modelling. Several other sensitivities will be pointed out in (c), namely those coming from Q values, which for heavy nuclei often have to be calculated using global structure models that extrapolate from known nuclei. This is the case particularly in the r-process, where there are shell closures and waiting points in nuclei far from stability, which lead to peaks in predicted abundances.

Finally (d) we describe the indirect methods that can be used to determine the properties of nuclei that are relevant to astrophysical reactions that can hardly or never be measured directly. Among these methods are transfer, charge exchange and break up reactions, using radioactive beams produced in laboratory accelerators.

Part IV: Indirect Measurements

11. **Accelerator Experiments**: In this chapter a brief introduction to the new radioactive nuclear beam (RNB) facilities is presented. In (a) the characteristics of the major RNB facilities are summarized, both the high energy laboratories (such as GSI, GANIL, NSCL, RIKEN) and the low energy facilities. We include a brief discussion of the accelerator process used in each case and the limiting factors determining the properties of the generated primary or secondary beam. Given that most reactions with exotic beams performed today involve inverse kinematics, the transformation from normal to inverse kinematics is derived (b), and the characteristics of the experimental observables for inverse kinematics are compared to those of normal kinematics. In (c) a short discussion of the most frequent detection techniques are presented, in particular the neutron time of flight (TOF) method.

12. **Spectroscopy using two body reactions**: This chapter brings together various elements introduced in part II, with the aim of extracting structure information about the radioactive nuclide.

Section (a) makes use of 4.b and 5.c to determine the single particle *l,j* structure and the spectroscopic factors from transfer data. The dependence on the optical potential and the adiabatic approximation for deuteron breakup in (d,p) or (p,d) are explicitly discussed. Also the application of transfer reactions in astrophysics using the Asymptotic Normalization Coefficient (ANC) method is considered.

Similarly, in (b), the reaction and structure aspects from 4.d and 5.e are applied to charge exchange reactions to extract Gamow-Teller strengths.

In (c), knock-out reactions are used to extract both (*l,j*) and spectroscopic factors. An outline of the Glauber techniques is included and further detailed in 14.c.

Higher-order corrections to transfer, such as coupled reaction channels (CRC) and inelastic excitations (within collective or single particle couplings) are also discussed. Specific examples that have been used by experimental groups are included.

13. **Breakup methods**: This chapter outlines the fundamentals of a theory for the breakup of two-body projectiles.

It begins (a) with a definition of the kinematics of three body final states, and a discussion of the interactions within this three-body system.

In (b), DWBA methods are applied to breakup, with a consideration of the various alternative forms for the operator.

As a standard method for including higher order effects, continuum bins are introduced in (c), and the continuum discretized coupled channel (CDCC) method is detailed in (d). For (b), (c) and (d) several examples of calculations are provided.

Depending on the experimental setup, different observables should be calculated. In (e) there is a discussion of breakup observables and the construction of coincidence cross sections is outlined.

The Trojan horse method (f) makes use of specific breakup reactions to determine capture rates at low energies for one fragment, and simplified models for these reactions are discussed.

In (g) we show how to analyse breakup data to extract structure information.

14. **Approximate methods**: This chapter describes several approximations of broad application in nuclear physics.

In (a) the concept of barrier tunnelling for fission and fusion is considered and WKB calculations are presented.

Eikonal approximations (b) are particularly suited for high energy reactions. Various eikonal models are discussed for elastic scattering and breakup, using potentials, the optical limit form, and few-body adiabatic models.

Also broadly used in Coulomb breakup reactions, first order semi-classical models will be overviewed in (c).

In (d), arguments are given concerning matching conditions for energy, momentum and angular momentum transfers in reactions, which give selectivity rules.

The spin-zero approximation (typically introduced to reduce the number of channels in the calculation) is presented in (e).

Part V: Further Topics

15. **Three-body nuclei**: Discussion of two-nucleon bound states: their structure, and reactions leading to and from them.

Section (a) defines three-body wave functions for two nucleons plus a core. This wave function is represented using shell-model, Jacobi and hyperspherical coordinates, along with the Hamiltonian and solution methods for each coordinate system. Their relative merits for deeply bound vs halo states, and references are given to methods and programs for calculation.

Transfer reactions to and from two-nucleon bound states may be approximated (b) as a di-nucleon cluster transfer, or calculated more microscopically with both simultaneous and sequential amplitudes interfering coherently. Numerical examples are given, with a brief discussion of non-orthogonality corrections.

Finally in (c) the topic of 3-body continuum states is introduced, and formulae for excitation and decay of such states given in terms of intermediate 2-body resonances (where these exist), or directly (using 3-body wave functions). Methods are discussed for capture reactions leading to 3-body bound (or quasi-bound) states via intermediate resonances, in particular those for $\alpha+2n\rightarrow {}^{6}He$, $2\alpha+n\rightarrow {}^{9}Be$ and $3\alpha\rightarrow {}^{12}C$, the importance of the last reaction being emphasized.

16. **Nuclear reactions involving electrons**: This chapter considers some influences of electron processes that influence nuclear astrophysical reactions.

It begins with a short summary of the standard approach to electron screening, both in stellar environments and in the laboratory. The basic theory (a) for electron capture and (b) for internal conversion reactions is described, including estimates for some key cross sections.

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